Experiments with Load Flow Pricing Models

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Abstract

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Spot pices of electricity are determined in New Zealand (and a number of other electricity markets in the the world) using a linear programming model to construct a dispatch schedule to meet metered loads at the nodes of the transmission network. The linear program seeks to minimise in each half hour the delivered cost of the energy, as represented by the prices that generators offer their power to the market, while accounting for transmission losses, network constraints and spinning reserve constraints. The ex-post electricity price at any given node of the transmission system is given by the shadow price of the energy balance constraint at optimality. We discuss the results of some experiments carried out with a small electricity pricing model developed in the Department of Engineering Science. The prices obtained from these linear programming models have some interesting properties. Some of these properties, although counterintuitive, have rational explanations. Other properties are less benign, and arise in circumstances when a linear programming model is an inappropriate approximation of the true load flow problem. We shall explain these effects, and discuss some possible remedies.

1 Introduction

The spot market for wholesale electricity was introduced in New Zealand on October 1, 1996, as part of an ongoing restructuring of electricity generation and distribution. Electricity cannot be stored, and when it is generated it must be distributed through a transmission network to the locations at which it is required. By separating the generation function from the transmission and distribution functions, electricity markets endeavour to stimulate competition amongst suppliers to deliver power to consumers at a competitive price, while providing sensible signals for investment and new entry. In recent years electricity markets of varying forms have emrged in most western countries, and their study has led to a substantial body of research into electricity market design (see e.g. [2]).

The New Zealand choice of market design was a nodal pricing model in which spot prices of electricity are determined for each half hour at approximately 470 buses (i.e. nodes) of the transmission network. Spot pices of electricity are determined using a linear programming model (called Scheduling, Pricing and Dispatch Software or SPD) to construct a dispatch schedule to meet metered loads at the nodes of the transmission network. The linear program seeks to minimise in each half hour the delivered cost of the energy, as represented by the prices that generators offer their power to the market, while accounting for transmission losses, network constraints and spinning reserve constraints. The ex-post electricity price at any given node of the transmission system is given by the shadow price of the energy balance constraint at optimality.

In this paper we are concerned with the properties of the dispatch and the nodal prices that are yielded by this process. We shall focus on the mathematical properties of linear programming models, and endeavour to explain some of the problems that these features can lead to when applied to optimal load flow models. We make no claim to the originality of our analysis; many of the features of SPD that we discuss in this paper are well known to users, and are openly discussed by New Zealand market participants (see [4]). However, it is hoped that this paper can lead to some understanding of the drawbacks in using linear programming models, and act as a guide to improving pricing and dispatch software to overcome these problems if they are deemed to be worth correcting.

The paper is laid out as follows. In the next section we show how a linear programming model of optimal power flow arises from approximating the equations of AC power flow, and then describe in theoretical terms how this approximation might lead to paradoxical or pathological results. In Section 3, we describe a simple pilot model of the New Zealand transmission network that we have developed in the Department of Engineering Science. This model is used in Section 4 to test whether the paradoxical behaviour described in Section 2 can actually occur in reasonably realistic circumstances.

2 Optimal Power Flow

The software (SPD) used to dispatch and price power in New Zealand is based on a DC-load flow model of electricity transmission. This model ignores reactive power, and approximates the real power flows by formulae that are analogous to Kirchhoff's formulae for direct current (DC). In this section we derive these equations to illustrate how this approximation occurs. (Since it is rather technical, this section may be skipped by the reader without losing the main points of the paper.)

Consider a transmission system represented by a network of nodes and lines. Let us consider a typical line joining nodes k and l. This line has an impedance $z = r + jx$, and a resulting admittance $y = \frac{1}{z} = G + jB$, where for notational convenience we suppress the dependence of the parameters on k and l. Suppose that we inject some power $P^s + jQ^s$ into node k for transmission to node l. Here P is known as the *active* power and Q is the *reactive* power. If the phasor of current from k to l is I, then we have

$$P^s + jQ^s = V_k I^*,$$

where V_k is the voltage phasor ($v_k e^{j\theta_k}$) at node k and I^* is the complex conjugate of the current phasor. The power which arrives at node l is $P^r + jQ^r$ defined by

$$P^r + jQ^r = V_l I^*.$$

Given the admittance $G + jB$ of the line we can compute

$$I = (v_k e^{j\theta_k} - v_l e^{j\theta_l})(G + jB),$$

giving

$$I^* = (v_k e^{-j\theta_k} - v_l e^{-j\theta_l})(G - jB),$$

and

$$\begin{aligned} P^s + jQ^s &= v_k^2 G - v_k v_l \cos(\theta_k - \theta_l) G - v_k v_l \sin(\theta_k - \theta_l) B \\ &\qquad + j(-v_k^2 B - v_k v_l \sin(\theta_k - \theta_l) G + v_k v_l \cos(\theta_k - \theta_l) B), \end{aligned} \tag{1}$$

$$\begin{aligned} P^r + jQ^r &= -v_l^2 G + v_l v_k \cos(\theta_l - \theta_k) G + v_l v_k \sin(\theta_l - \theta_k) B \\ &\qquad + j(v_l^2 B + v_l v_k \sin(\theta_l - \theta_k) G - v_l v_k \cos(\theta_l - \theta_k) B). \end{aligned} \tag{2}$$

In the New Zealand electricity market, there are currently no prices computed for reactive power. In SPD this entails ignoring the imaginary parts of (1) and (2) to yield

$$P^s = v_k^2 G - v_k v_l \cos(\theta_k - \theta_l) G - v_k v_l \sin(\theta_k - \theta_l) B \tag{3}$$

$$P^r = -v_l^2 G + v_l v_k \cos(\theta_l - \theta_k) G + v_l v_k \sin(\theta_l - \theta_k) B. \tag{4}$$

For large, high voltage transmission systems we can usually assume that the voltage magnitude does not change much from node to node. In the DC-load flow model we assume that the voltage magnitude is the same at each node, so (by scaling the problem appropriately) we may assume that $v_k = 1.0$, for every node k. The DC-load flow model also requires that the phase angle differences over transmission lines are small. In these circumstances $\sin(\theta_k - \theta_l)$ is approximately equal to $(\theta_k - \theta_l)$, and $\cos(\theta_k - \theta_l)$ is approximately equal to $1 - \frac{(\theta_k-\theta_l)^2}{2}$. This results in the following expressions:

$$P^s = \frac{(\theta_k - \theta_l)^2}{2}G - (\theta_k - \theta_l)B, \tag{5}$$

$$P^r = -\frac{(\theta_k - \theta_l)^2}{2}G - (\theta_k - \theta_l)B. \tag{6}$$

It is instructive to study equations (5) and (6). If the phase angle differences over transmission lines are so small that $(\theta_k - \theta_l)^2$ is negligible then $P^s = P^r = -(\theta_k - \theta_l)B$. This approximation gives a real power flow in the line of $-(\theta_k - \theta_l)B$, which is a form of Ohm's Law with phase angles taking the role of DC voltages, and $(-1/B)$ taking the role of DC line resistance. In this model there are no power losses. If we choose not to ignore terms in $(\theta_k - \theta_l)^2$ then the power P^s sent exceeds the power P^r received by an amount equalling $(\theta_k - \theta_l)^2 G$. Denoting $-(\theta_k - \theta_l)B$ by p we obtain

$$P^s = p + \frac{p^2}{2B^2}G, \tag{7}$$

and

$$P^r = p - \frac{p^2}{2B^2}G. \tag{8}$$

Since p is the lossless approximation to the DC-load flow, these equations indicate that the sending power at one end of the line must be increased by half the line losses $(\frac{p^2}{2B^2}G)$ to deliver at the other end the lossless power decreased by half the line losses. The line losses are proportional to p^2, the square of the lossless power flow.

Now we are in a position to define a model to compute an optimal dispatch and nodal prices. Consider a network of n nodes with given active power loads and known generation costs. Let y_k denote a level of injection of active power at node k, and let d_k be the load at node k. We seek to minimize the revealed cost $\sum_k C_k(y_k)$ of power injection at the nodes, subject to meeting the loads. At each node k in the New Zealand market $C_k(y_k)$ is a piecewise linear convex function, with a derivative represented by the aggregated offer stack of all participants at node k.

The line from k to l is denoted by (k, l), where we adopt the convention that $k < l$ and allow negative power flows. Each line has a capacity U_{kl} giving the maximum power flow that can be sent along that line. This gives

$$-U_{kl} \leq P^s_{kl} \leq U_{kl}. \tag{9}$$

Power flow must be conserved, yielding

$$\sum_{l=1}^{k-1} P^r_{lk} - \sum_{l=k+1}^{n} P^s_{kl} + y_k = d_k, \tag{10}$$

and for every $k < l$ we must have equations relating P^r_{kl} to P^s_{kl}. There are several ways to do this depending on how we choose to approximate the losses.

First with no losses we obtain

$$P^s_{kl} = P^r_{kl} = -(\theta_k - \theta_l)B_{kl}. \tag{11}$$

In these circumstances minimizing $\sum_k C_k(y_k)$ subject to (9), (10) and (11) is a convex nonlinear program, for which a globally optimal solution can be obtained using a nonlinear programming package (see e.g.

[3]). In the special case where $C_k(y_k)$ is a piecewise linear convex function, the convex program becomes a linear program. At first sight this has the appearance of a single-commodity minimum cost network flow problem, a well-known model in operations research. Observe, however, that the constraint (11) for each line (k,l) imposes a restriction on the power flow in that line, namely that it is driven by a difference in voltage angles. This means that for any loop L in the network we have

$$\sum_{(k,l)\in L} \frac{P^s_{kl}}{B_{kl}} = \sum_{(k,l)\in L} -(\theta_k - \theta_l) = 0.$$

The important implication is that adding loops to the transmission system adds constraints to be satisfied with potential increases in costs, and important consideration to be borne in mind when considering investments in new lines.

If we increase the accuracy of the approximation to *quadratic* losses we obtain

$$p_{kl} = -(\theta_k - \theta_l)B_{kl}, \tag{12}$$

$$P^s_{kl} = p_{kl} + \frac{G_{kl}}{2B^2_{kl}} p^2_{kl}, \tag{13}$$

$$P^r_{kl} = p_{kl} - \frac{G_{kl}}{2B^2_{kl}} p^2_{kl}. \tag{14}$$

In these circumstances minimizing $\sum_k C_k(y_k)$ subject to (9), (10), (12), (13), (14) is a nonlinear program. However even though $\sum_k C_k(y_k)$ is a convex function, and the constraints contain convex functions (quadratics) the resulting optimisation problem is not guaranteed to be convex. (A convex optimisation problem has a convex objective function and a convex feasible region – the nonlinear program here is not convex beacuse its feasible region is not convex.) It is possible to show that if we replace (10) by

$$\sum_{l=1}^{k-1} P^r_{lk} - \sum_{l=k+1}^{n} P^s_{kl} + y_k \geq d_k, \tag{15}$$

(so excess power can be shed at any node with no penalty) then the feasible region becomes convex, and so the optimal load flow problem becomes convex.

A more detailed model might treat active power with *full* losses. Here we allow the nodal voltage magnitudes to vary between specified bounds,

$$1 - \epsilon \leq v_k \leq 1 + \epsilon, \tag{16}$$

and in place of (9–14) we use the full active power-flow equations:

$$P^s = v_k^2 G - v_k v_l \cos(\theta_k - \theta_l)G - v_k v_l \sin(\theta_k - \theta_l)B \tag{17}$$

$$P^r = -v_l^2 G + v_l v_k \cos(\theta_l - \theta_k)G + v_l v_k \sin(\theta_l - \theta_k)B. \tag{18}$$

In this model we lose convexity entirely, even if (15) is used.

In the above discussion we have emphasized convexity as an important feature of any mathematical optimisation models we might consider for computing optimal dispatch. Convex nonlinear programs have nice properties. First one can guarantee that any solution procedure that yields a local optimum (one that is best if one only considers small perturbations of the variables from their curent values) will also yield a global optimum (one that is best for all possible choices of the variables). A second important property of convex optimisation problems is that they can be approximated by linear programs. The error in this approximation can be made arbitrarily small by making the linear programming problem have enough variables and constraints. To obtain globally optimal solutions to optimisation problems that are not convex requires very sophisticated techniques. These problems can be approximated by mixed integer linear programming problems, but the calculation time for these models becomes prohibitive as the size increases, so it may be impossible to obtain an answer with a desired level of accuracy. The third important property of convex optimisation problems is that they have a duality structure. The variables of the dual problem often have an interpretation as marginal prices. It is these variables that give the nodal prices reported by SPD.

The SPD software used in the New Zealand spot market is described in detail in [1]. It approximates quadratic losses by a piecewise-linear function, and then applies a linear programming algorithm. As stated in [1] this approximation is based on the premise that optimisations naturally favour reducing losses. This implies that excess power can be shed at any node without penalty, a sufficient condition for convexity. In our notation, the linear programming approximation in SPD replaces p_{kl} by the difference of two nonnegative power flows, to give

$$p_{kl} = \sum_{i=1}^{m} u_{kl}^{(i)} - w_{kl}^{(i)}, \qquad 0 \leq u_{kl}^{(i)}, w_{kl}^{(i)} \leq b_{kl}^{(i)}, \qquad i = 1, 2, \ldots, m,$$

and approximates the quadratic terms in (13) and (14) to give

$$P_{kl}^{s} = \sum_{i=1}^{m} u_{kl}^{(i)} - w_{kl}^{(i)} + \frac{G_{kl}}{2B_{kl}^{2}} \sum_{i=1}^{m} a_{kl}^{(i)}(u_{kl}^{(i)} + w_{kl}^{(i)}),$$

$$P_{kl}^{r} = \sum_{i=1}^{m} u_{kl}^{(i)} - w_{kl}^{(i)} - \frac{G_{kl}}{2B_{kl}^{2}} \sum_{i=1}^{m} a_{kl}^{(i)}(u_{kl}^{(i)} + w_{kl}^{(i)}),$$

where the slopes $a_{kl}^{(i)}$ and breakpoints $b_{kl}^{(i)}$ are chosen to make $\sum_{i=1}^{m} a_{kl}^{(i)}(u_{kl}^{(i)} + w_{kl}^{(i)})$ a good approximation of p_{kl}^2.

One disadvantage of such a formulation is that artificial price breaks are produced at breakpoints of the piecewise linear loss functions, which do not appear in the (more natural) quadratic loss formulation. On the other hand, the piecewise linear approximations become more accurate as more pieces are used, and the inaccuracy in the solution seems to be small, even with only a few breakpoints in the loss functions. An advantage of using piecewise linear losses is that an approximation to the optimal power flow can be found by reliable and efficient commercial software for linear programming.

In the next section we study a simple pricing and dispatch model of the New Zealand transmission system under four different modelling assumptions. We begin by approximating the dispatch problem as a linear program, under the quadratic loss assumption. The dangers of assuming convexity in this situation are identified. We digress to consider overcoming the convexity issue using mixed integer programming. The third model we consider uses a quadratic loss assumption, but does not assume convexity. Finally we compare these models with an active power model assuming full losses.

3 Experiments

To illustrate the issues outlined above we have created a small (7-node) DC-load flow model of the New Zealand transmission grid. The network for this is shown in Figure 1.

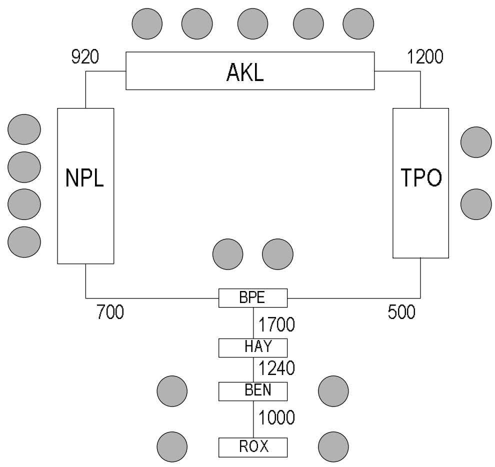

Figure 1: Experimental Network.

There is a load to be satisfied at each bus, by a total of 17 generators. The figure shows at each bus a set of generators that offer generation at this bus. The line data that we have used in this model is given in the following table.

Line	Capacity (MW)	G	B	$\frac{G}{2B^2}$
NPL-AKL	920	270	-1622	0.000051
TPO-AKL	1200	1538	-12308	0.000005
BPE-NPL	700	7376	-31612	0.000004
TPO-BPE	500	8	-909	0.000005
HAY-BPE	1700	2193	-13338	0.000006
BEN-HAY	1240	-	-	-
ROX-BEN	1000	165	-764	0.000141

Table 1: Line data: base case

These numbers have been computed to be approximations to the actual values for lines in the New Zealand transmission network so as to illustrate the effects that we wish to study. We must be careful, however, that this model is not used to draw conclusions about the magnitude or the frequency of these effects. This would require a study with a full implementation of SPD.

Example 1: Base Case

The first example illustrates the difference in solution obtained with different approximations used for the line losses. The base model uses the data in the following tables.

Node	Demand (MW)
AKL	1179.00
NPL	173.00
TPO	1260.71
BPE	172.84
HAY	559.15
BEN	687.62
ROX	831.80

Table 2: Node demands: base case

Station	Node	Quantity (MWHr)	Price ($/MWHr)
HLYA	AKL	490	25
HLYB	AKL	490	26
MDN	AKL	120	200
OTA	AKL	90	90
SDN	AKL	110	99
NPLA	NPL	230	30
NPLB	NPL	345	31
SFD	NPL	208	36
TCC	NPL	340	99
WKO	TPO	600	27
GEO	TPO	257	1
WHI	BPE	162	140
NIO	BPE	550	24
WTK	BEN	1740	20
SIO	BEN	100	1
ROX	ROX	800	9
MAN	ROX	590	8

Table 3: Generator offers: base case

The nodal prices obtained under each of the four loss models is shown in Table 9. We set $\epsilon = 0.05$ in the full model, so nodal voltage magnitudes can vary by 5% from 1.0 p.u. In this example the quadratic model provides ta good approximation to the losses in the full loss model, with the piecewise linear model improving as the number of steps in the piecewise linear loss functions increases.

Node	Full losses	Quadratic losses	Piecewise losses (3 steps)	Piecewise losses (10 steps)
AKL	29.63	29.87	31.36	29.39
NPL	27.48	27.56	29.31	27.30
TPO	29.73	29.99	31.30	29.46
BPE	27.27	27.33	29.09	27.08
HAY	27.01	27.04	28.79	26.81
BEN	20.00	20.00	20.00	20.00
ROX	14.21	14.88	15.34	14.72

Table 4: Nodal prices under various loss models in Example 1

Example 2: Non-convex optimal dispatch problem

In this example we illustrate some of the drawbacks of using a piecewise linear model for losses when the problem is not convex. This problem has been studied in situations where generators (who might pay to be dispatched)are allowed to offer tranches with negative prices (see [4]). Here the incentive for SPD is to maximise the payments of such generators, so it seeks to maximise the losses in order that more of this power can be dispatched. When maximising losses, SPD assigns flows to links in the model to produce a solution that solves the linear program, but cannot be implemented in the physical network. Symptoms of this behaviour are solution variables with both $p_{kl} > 0$ and $p_{lk} > 0$, or $u_{kl}^{(m)}$ being positive (at a high loss factor) while $u_{kl}^{(1)} = u_{kl}^{(2)} = \ldots = u_{kl}^{(i)} = 0$ for some $i < m$.

In fact SPD can fail to give a physically implementable solution, even in the absence of negative offers. We illustrate this in the example by breaking the link between BPE and HAY and changing the capacities of the links (AKL,NPL) to 2000, and (BPE,NPL) to 800, while setting the offer prices at NPL to be zero as shown in Table 5.

The solutions delivered by the full-loss model and the quadratic loss model prices were both physically realisable. However as shown by Table 6 the link (BPE-NPL) contained flow in opposite directions when the piecewise linear model is used.

Station	Node	Quantity (MWHr)	Price ($/MWHr)
NPLA	NPL	230	0
NPLB	NPL	345	0
SFD	NPL	208	0
TCC	NPL	340	0

Table 5: Generator offers at New Plymouth for Example 2

Link	Full losses	Quadratic losses	Piecewise losses (3 steps)	Piecewise losses (10 steps)
NPL-BPE	746.61	676.198	800.00	800.00
BPE-NPL	0.00	0.00	120.36	119.43

Table 6: Sent power under various loss models for Example 2

The prices at the nodes are given by Table 7. It is interesting to observe in this example that the nodal prices do not seem to be affected by the non-physical dispatch.

Node	Full losses	Quadratic losses	Piecewise losses (3 steps)	Piecewise losses (10 steps)
AKL	25.00	25.00	25.00	25.00
NPL	0.00	0.00	0.00	0.00
TPO	27.43	27.96	27.99	27.98
BPE	-0.99	-1.16	-1.18	-1.16
HAY	23.56	23.56	28.79	23.29
BEN	20.00	20.00	20.00	20.00
ROX	14.21	14.88	15.33	14.72

Table 7: Nodal prices under various loss models for Example 2

Example 3: Loop constraints

In this example we study the effect of the loop constraint

$$\sum_{(k,l)\in L} \frac{P^s_{kl}}{B_{kl}} = 0 \tag{19}$$

for L defined by AKL–NPL–BPE–TPO. We retain all the data from Example 2, but break the (AKL,TPO) link. This means that the constraint (19) is removed from the optimisation problem, enlarging the set of possible optimal solutions. As a consequence the cost of meeting the load decreases as shown below in Table 8.

Generation Cost	Full losses	Quadratic losses	Piecewise losses (3)	Piecewise losses (10)
With (AKL,TPO)	46082	49074	50546	48817
Without (AKL,TPO)	41380	41395	42857	48761

Table 8: Nodal prices under various loss models for Example 3

The nodal prices with the link removed are as follows.

Node	Full losses	Quadratic losses	Piecewise losses (3 steps)	Piecewise losses (10 steps)
AKL	-	-	0.00	-
NPL	-	-	0.00	0.00
TPO	-	-	27.00	27.00
BPE	-	-	0.00	0.00
HAY	-	-	28.79	23.29
BEN	-	-	20.00	20.00
ROX	-	-	15.34	14.72

Table 9: Nodal prices under various loss models for Example 3

Example 4: Nonunique shadow prices
In this example we look at the shadow prices in the piecewise linear model (to be completed)

Example 5: Nonunique shadow prices
In this example we examine the shadow prices in the quadratic model (to be completed)

4 Discussion

There has been considerable discussion of problems with non-physical dispatch. Formulating the dispatch problem as a mixed integer program can prevent non-physical dispatch. However, solving this can be very slow, and can lead to incorrect dual variables. Dual variables might not be unique even with smooth loss functions.

References

[1] Alvey, T., Goodwin, D., Ma, X., Streiffert, D. and Sun, D. 1998. A security-constrained bid-clearing system for the New Zealand wholesale electricity market *IEEE Transactions on Power Systems*, 13:340–346.

[2] Chao, H-P. and Huntingdon, H.G. 1998. *Designing Competitive Electricity Markets* Kluwer Academic, Boston.

[3] Murtagh, B.A. and Saunders, M.A. 1987. *Minos 5.1 Users Guide*, Technical Report SOL 83-20R, Systems Optimization Laboratory, Stanford University.

[4] Woods, M. and Sise, G. Negative Offers and Prices in SPD. Report for the Market Pricing Working Group of NZEM. http://www.m-co.co.nz/h9papers/markPrice/9904/NegativeOffers.doc