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On Financial Transmission Rights in Electricity Pool Markets

Andy Philpott and Geoffrey Pritchard

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1 Introduction

Electricity pool markets based on nodal pricing have emerged in a number of regions of North and South America, Australia, New Zealand, and the Nordic countries. In an electricity pool with a single node, all market participants trade (at any one point in time) at a single system marginal price. In a full nodal pricing model, the price depends on location. In the absence of constraints and losses the price at every node is identical to the system marginal price. In practice thermal constraints on the power flow in the lines, and power losses due to line impedance, result in a set of prices that varies with location. The variation of electricity spot prices with location in these markets has resulted in the development of market instruments to hedge the volatility in these price differences.

Financial Transmission Rights (FTRs) are one approach to solving this problem. An FTR is a financial instrument held by a market participant that pays an income stream based on the nodal prices observed in the transmission system over the course of the contract. An FTR can be specified by a vector of nodal loads and injections (where the loads are taken as positive) and the coupon payment at any time is equal to the inner product of the nodal price vector with the FTR. (In practice most FTRs involve only two nodes.) FTRs were first proposed by Hogan [8], and have received a lot of attention in the literature under various names (they are called fixed transmission rights in the PJM market, transmission congestion contracts or TCCs in New York, and financial congestion contracts, or FCCs in New England.)

Our purpose in this paper is to provide an overview of some of the features of these contracts, as they apply to markets that are dispatched as DC-load flow networks with constraints and convex line losses. We shall review some of the important theoretical contributions in this area, generalising them where possible, and examine using examples some of the effects on participant behaviour that one might expect to see in the presence of these contracts.

One approach to investigating these effects is to model electricity markets as a Nash-Cournot game in which each generator offers a single quantity of energy assuming zero conjectural variations. It is possible to construct Nash equilibria for these models, and use these to investigate the effects of FTRs on agent behaviour. By restricting attention to single offers facing a known elastic demand function, the Nash-Cournot model has some deficiencies in representing the true situation, in that generators in most pool markets typically offer supply functions rather than single offers, and demand is uncertain. One approach to reflect the first of these features of pool markets is to optimize over a parametrized family of supply functions (see [7] for a linear supply function model of this type.) However, as observed in [3] the real advantage of supply function offers in practice comes from their ability to optimize over a range of uncertain demands, and this flexibility will only be represented for high-dimensional parametrizations resulting in considerable computational effort. On the other hand the construction of general supply-function equilibria in a network setting seems to be very difficult.

A compromise is to use the market distribution approach of Anderson and Philpott [4], which treats competitors as if they offered supply functions drawn from a suitable probability distribution, called the market distribution. Since demand shocks can be modelled as vertical shifts in a competitor's supply function, the market distribution can be constructed so that it also incorporates uncertainty in demand. We present several examples to illustrate the power of the market distribution approach in evaluating the effects of FTRs on competitor behaviour.

The paper is laid out as follows. In the next section we extend the revenue adequacy result proved by Hogan for smooth problem data to a general convex programming setting. This admits as a special case convex dispatch models with convex piecewise linear losses and piecewise linear benefit functions, as used in the New Zealand dispatch software SPD (see e.g. [4]). We show by example how revenue adequacy can fail for models that are not convex. Section 3 discusses the effect of FTRs on generators who already have some degree of market power. We show by example that FTRs may encourage the use of this power or discourage it depending on the location of the generator in the network. These examples mirror those described by Joskow and Tirole [11]. Section 4 discusses the potential of FTRs to encourage or discourage investments in the transmission network. We conclude the paper with a general discussion in Section 5.

2 Revenue Adequacy

In this section we discuss the funding of FTRs from rentals accrued by the Independent System Operator (the ISO) in a wholesale pool electricity market. A well-known result proved by Hogan states that such revenue adequacy is guaranteed by the FTRs being *simultaneously feasible* for the network constraints, in the sense that they can be dispatched through the network without exceeding

capacities. This result was first proved by Hogan [8] for lossless networks, extended to quadratic losses by Bushnell and Stoft [5], and further generalised to smooth nonlinear constraints by Hogan [9]. The result is a general consequence of the convexity of the dispatch problem.

We formulate the dispatch problem as the following convex optimisation problem in a transmission network with n nodes.

$$\begin{array}{ll} \text{P:} & \text{minimise} \quad \sum_i \sum_{j \in O(i)} c_j x_j \\ & \text{subject to} \quad g_i(f) + \sum_{j \in O(i)} x_j - z_i = d_i, \quad i = 1, 2, \dots, n \\ & \quad \quad \quad z_i \geq 0, \quad i = 1, 2, \dots, n \\ & \quad \quad \quad x \in X \\ & \quad \quad \quad f \in U \end{array}$$

Here d_i is the demand at node i , and the first set of constraints represent conservation of flow at the nodes. We assume here that the problems associated with negative prices in situations of low demand do not occur, and so that at optimality the flow balance constraints are satisfied as equations. This means that we can represent the flow balance equations as inequalities, without losing any physical realism. (See below for further comment on this.) In the formulation x_j is the level of dispatch of tranche $j \in O(i)$ where $O(i)$ is the set of tranches offered at node i , and each tranche $j \in O(i)$ is offered at price c_j . We require that x lies in the convex set X , which defines the tranche levels. Although the problem P assumes (as in most electricity pool markets) that generators offer supply curves that are step functions in what follows the analysis will hold for any increasing supply function.

In the model P f is a vector of branch flows. The function $g_i(f)$ is a general concave function giving the amount of power flow entering node i when the link flows are f . Approximations based on DC load flow define $g_i(f)$ to be a concave quadratic function, meaning that branch losses are a quadratic function of power flow. Alternatively a linear programming representation of P treats $g_i(f)$ as a concave piecewise linear function. Both of these models (as well as a model without losses) are special cases of the general framework we use. We require the vector of flows f to lie in the convex set U , which represents any flow bounds on f as well as any electrical constraints (such as loop flow constraints) that the flows must satisfy.

Now suppose that (x^*, f^*, z^*) solves P and that P satisfies a constraint qualification (e.g. P has a feasible solution with $z_i > 0$). Since P is a convex program it satisfies the Lagrangian Duality Theorem (see e.g [12]). This states that there is a set of optimal Lagrange multipliers π (the nodal prices for the optimal dispatch) such that (x^*, f^*, z^*) minimises the Lagrangian

$$\mathcal{L}(x, f, z) = \sum_i \sum_{j \in O(i)} c_j x_j + \sum_i \pi_i (d_i - g_i(f) - \sum_{j \in O(i)} x_j + z_i)$$

over $z \geq 0$, $x \in X$, $f \in U$. We use this to prove the following lemmas.

Lemma 1 $\pi_i \geq 0$ and $\pi_i(g_i(f^*) + \sum_{j \in O(i)} x_j^* - d_i) = 0$

Proof. If $\pi_i < 0$, then for any feasible z , $\pi_i z_i \leq 0$, and so $\mathcal{L}(x, f, z)$ is unbounded below over $z \geq 0$. Since $\mathcal{L}(x, f, z)$ has a minimum by the Lagrangian Duality Theorem, it follows that $\pi_i \geq 0$. Now the minimising choice of z_i^* must make $\pi_i z_i^* = 0$. But since by feasibility

$$z_i^* = g_i(f^*) + \sum_{j \in O(i)} x_j^* - d_i,$$

this entails the result. ■

Lemma 2 The rental earned by the ISO is $\sum_i \pi_i g_i(f^*)$

Proof. The ISO rental is equal to the difference between what it is paid by loads and what the ISO pays generators. Formally this is

$$\sum_i \pi_i d_i - \sum_i \pi_i \sum_{j \in O(i)} x_j^* = \sum_i \pi_i g_i(f^*)$$

by virtue of Lemma 1. ■

Lemma 3 For every $f \in U$, $\sum_i \pi_i g_i(f^*) \geq \sum_i \pi_i g_i(f)$

Proof. Since (x^*, f^*, z^*) minimises $\mathcal{L}(x, f, z)$, f^* should be chosen in U to minimise $-\sum_i \pi_i g_i(f)$ (this is the only term in $\mathcal{L}(x, f, z)$ that contains f). So for all $f \in U$,

$$\sum_i \pi_i g_i(f^*) \geq \sum_i \pi_i g_i(f).$$

■

Now let us consider a situation in which there are A extant FTR contracts described by vectors $h(\alpha)$ for $\alpha = 1, \dots, A$. The contract with vector $h(\alpha)$ pays its holder $\sum_i \pi_i h_i(\alpha)$ once the dispatch problem has been solved. (This is a fairly general notion of FTR; it includes the common "balanced" FTR (in which there are nodes i_1, i_2 with $h_{i_1}(\alpha) = -h_{i_2}(\alpha)$ and $h_j(\alpha) = 0$ for all $j \notin \{i_1, i_2\}$) and "spot" FTR (in which there is a node i_1 with $h_j(\alpha) = 0$ for all $j \neq i_1$) as well as other types.) Suppose that these are simultaneously feasible in the sense that there exist y, z with

$$\begin{array}{rcl} \text{SF:} & g_i(y) - z_i & = \sum_{\alpha} h_i(\alpha), \quad i = 1, 2, \dots, n \\ & z_i & \geq 0, \quad i = 1, 2, \dots, n \\ & y & \in U. \end{array}$$

That is, $\sum_{\alpha} h(\alpha)$ represents a vector of injections and offtakes which may be "dispatched through the grid". Note that it is permissible to shed power at nodes (i.e. to have $z_i > 0$).

Theorem 4 *When the extant FTR contracts are simultaneously feasible as above, the rentals earned by the ISO are sufficient to fund the coupon payments to the FTR holders.*

Proof. The total coupon payment to be made is $\sum_i \pi_i \sum_\alpha h_i(\alpha)$, which for some y, z may be written

$$\begin{aligned} \sum_i \pi_i \sum_\alpha h_i(\alpha) &= \sum_i \pi_i (g_i(y) - z_i) \\ &\leq \sum_i \pi_i g_i(y), \end{aligned}$$

since $z_i \geq 0$, and $\pi_i \geq 0$. Now by virtue of Lemma 3, since $y \in U$, the right-hand side is bounded above by the rental from the actual dispatch, giving

$$\sum_i \pi_i \sum_\alpha r_\alpha h_i(\alpha) \leq \sum_i \pi_i g_i(f^*).$$

■

Lemmas 1–4, while mathematically true, have practical significance only when $z^* = 0$, since only then does the optimum (x^*, f^*, z^*) of P correspond to a physically implementable solution. In situations where $z^* \neq 0$, we must consider the true dispatch problem:

$$\begin{array}{ll} \text{NP:} & \text{minimise} \quad \sum_i \sum_{j \in O(i)} c_j x_j \\ & \text{subject to} \quad g_i(f) + \sum_{j \in O(i)} x_j = d_i, \quad i = 1, 2, \dots, n \\ & \quad \quad \quad x \in X \\ & \quad \quad \quad f \in U. \end{array}$$

This is the same as problem P, except that we have forced all $z_i = 0$. It is the “true” dispatch problem in the sense that it represents the problem of finding the best (in the economic sense) course of action from among all physically realisable courses of action.

Problem P is a relaxation of NP; allowing $z_i > 0$ adds some physically unrealisable dispatches to the feasible set. The key difference is that P is a convex optimization problem, while NP is in general non-convex.

It is common to replace NP by P, which being convex is computationally much easier to solve. However, it should be emphasized that whenever the optima of P and NP differ, it is the optimum of NP that is the correct one (while the optimum of P will not even be physically implementable). Such situations are associated with negative nodal prices.

Note, however, that there is no “non-convex” counterpart to the simultaneous feasibility condition SF. If the optimum (x^*, f^*, z^*) of P happens to have $z^* = 0$ (or equivalently, if the optimum of NP also happens to be optimal for P) then by Lemma 4 the corresponding ISO rentals will be adequate to fund any collection of FTRs that are simultaneously feasible in the sense of SF. But if this is not the case, then there is no simple condition (in the knowledge of

these authors) on the FTRs that will imply revenue adequacy. In particular, the condition that there exists y with

$$\text{NSF:} \quad \begin{array}{lcl} g_i(y) & = & \sum_{\alpha} h_i(\alpha), \quad i = 1, 2, \dots, n \\ y & \in & U. \end{array}$$

(i.e. $\sum_{\alpha} h(\alpha)$ may be “dispatched through the grid”, with no power shed at nodes) is insufficient to imply revenue adequacy when $z^* \neq 0$. This is explored in the following subsection, which presents examples where $z^* \neq 0$.

2.1 Revenue inadequacy caused by negative nodal prices

This section demonstrates that simultaneous feasibility may fail to guarantee revenue adequacy for FTRs if a situation arises where nodal prices become negative, due either to negatively-priced offers or to transmission constraints.

Example 5 *Revenue inadequacy created by negative offer prices*

Consider a network of two nodes joined by a single transmission line of capacity 1 and quadratic loss coefficient $r > 0$ (so that if an amount f of power is injected at one end of the line, $f - rf^2$ may be extracted from the other). This network can support (in the sense of simultaneous feasibility) one unbalanced FTR with injection 1 at one end of the line and offtake $1 - r$ at the other.

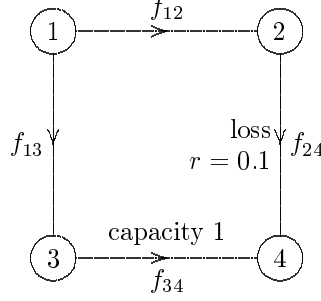
Now suppose that when the dispatch problem NP is solved, we have a generator G1 offering to inject quantity 1 at a negative price p_1 at one node (node A, say), while another generator G2 offers to inject quantity 1 at a negative price p_2 at node B. Suppose $p_1 < p_2 < 0$, and that there is a load of 1 at node B.

It is not hard to see that in the optimal dispatch, G1 generates 1 while G2 generates r . The nodal prices are $\pi_A = p_2(1 - 2r)$ and $\pi_B = p_2$, while the rental earned by the ISO is $(1 - r)\pi_B - \pi_A = rp_2$. (Note that this is negative, a situation possible only with negative nodal prices.)

However, if we have awarded the FTR with injection 1 at node B and offtake $1 - r$ at node A, then the coupon payment owing is $(1 - r)\pi_A - \pi_B = -p_2(3r - 2r^2)$. This is a positive amount, even though there are no rentals (negative rentals, in fact) to fund it with.

Example 6 *Revenue inadequacy created by a transmission constraint, with only positive offer prices.*

Consider the following four-node network:



All lines have unlimited capacity (except for the line shown with capacity 1) and are lossless (except for the line shown with quadratic loss coefficient $r = 0.1$). That is, the functions $g_i(f)$ in problem NP are

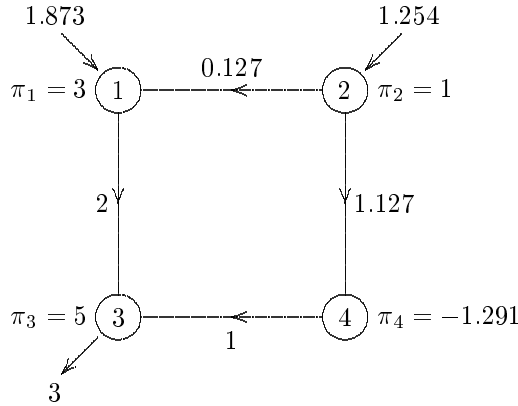
$$\begin{aligned}
 g_1(f) &= -f_{12} - f_{13} & g_2(f) &= \begin{cases} f_{12} - f_{24} & , f_{24} \geq 0 \\ f_{12} - f_{24} - r f_{24}^2 & , f_{24} \leq 0 \end{cases} \\
 g_3(f) &= f_{13} - f_{34} & g_4(f) &= \begin{cases} f_{24} - r f_{24}^2 + f_{34} & , f_{24} \geq 0 \\ f_{24} + f_{34} & , f_{24} \leq 0 \end{cases}
 \end{aligned}$$

In order to keep the loop flow constraint simple, we will suppose that the four lines have the same admittance, i.e. that the loop flow constraint is

$$f_{34} - f_{24} - f_{12} + f_{13} = 0.$$

This network can support (in the sense of simultaneous feasibility) one unbalanced FTR with injection 1.34109 at node 4 and offtake 1.32946 at node 3.

Now suppose that generation is offered (in unlimited quantity) at a price $p_1 = 3$ at node 1 and at a price $p_2 = 1$ at node 2. The only load is 3 at node 3. Solving problem NP yields the following optimal dispatch:



Note that the loop flow constraint has created a negative nodal price at node 4. The rental earned by the ISO is 8.127, which is less than the payment owing on the FTR of 8.379.

3 FTRs and Market Power

This section explores the consequences of awarding financial transmission rights to electricity market participants large enough to influence market prices.

It is well-understood that a generator with market power (i.e. a “price-maker”) will engage in strategic offering in order to maximize its profit. In general, this means offering supply at a price above marginal cost, or withholding some supply, or both, in order to increase the price at which electricity can be sold.

Intuitively, it is clear that ownership of FTRs by a price-maker will modify this behaviour, by increasing or decreasing the incentive to force up the price at the local node. Very simple economic models such as those to be found in [11] confirm that when an FTR over a line from an “upstream” location to a “downstream” one is awarded to a price-maker generator:

- If the generator is in the downstream location, its strategy will tend to become more “aggressive” (i.e. even further removed from the competitive strategy of offering all available supply at its marginal cost), as there is now even more incentive to force up the local price.
- If the generator is in the upstream location, its strategy will tend to become less aggressive – there is now less incentive to force up the local price.

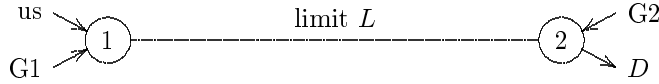
In a network with loop flow, either of the above effects is possible depending on the locations of the generator and the FTR. Finally, all of the above effects will be reversed if the FTR goes in the “wrong” direction (e.g. from a downstream node to an upstream one). (Such an FTR may seem unlikely, but could

arise if the FTR-holder made a wrong guess as to the direction of the flow on the line. For example, the line between the two islands of New Zealand usually carries the South Island’s abundant hydro-power northwards, but in a dry year may find itself constrained in the opposite direction.)

In the remainder of this section, we present two examples which illustrate in more detail how the problem of optimizing a price-maker’s offer curve is affected by the presence of FTRs. We use the market distribution function methodology of [4], wherein a general theory is presented for constructing a legal (i.e. monotone increasing) offer curve in response to uncertain demand and competitor behaviour. The uncertainty – and hence the need to consider complete offer curves rather than single offers – are important, since one motivation for the existence of FTRs is to allow uncertain price differences to be hedged.

Example 7 *FTR mitigating the effects of market power.*

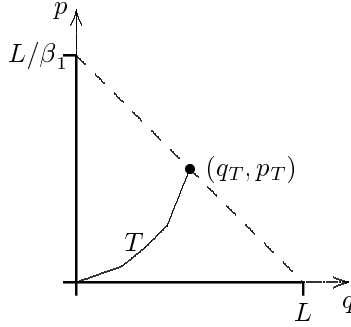
Consider a two-node grid in which “we” are the only generator with market power.



A competitive fringe consisting of many smaller generators and demand-side participants provides additional supply “G1” and “G2”; which is offered to the market via fixed aggregated offer curves. For simplicity, assume that these take the forms $q = \beta_1 p$ and $q = \beta_2 p$ (with $\beta_1 > 0$, $\beta_2 > 0$) at nodes 1 and 2 respectively. Demand D is located at node 2 and is random, with a probability density function f . The transmission line is lossless, but has a maximum capacity L . Suppose that we can generate power at no marginal cost, and that we own a balanced FTR for quantity q_f from node 1 to node 2.

The FTR means that we are effectively selling part of our output (a quantity q_f) at the node 2 price p_2 , rather than the node 1 price p_1 . Since we have less direct influence over p_2 than over p_1 , one might intuitively expect that the FTR will reduce our natural desire to force up p_1 by withholding supply. One might also expect that the line capacity constraint will be more likely to be reached, since our ownership of the FTR incentivizes us to create this very outcome. Both of these things turn out to be the case, as the following analysis shows.

We may be dispatched at a point (q, p) on the offer curve, with the line not at capacity, whenever $q + \beta_1 p < L$. This will be achieved for demand level $D = q + (\beta_1 + \beta_2)p$, since the two nodal prices will be equal. Alternatively, we may be dispatched at (q, p) , with the line at capacity, if $q + \beta_1 p = L$. This will require $D \geq L + \beta_2 p$, and the price at node 2 will then be $p_2 = \beta_2^{-1}(D - L)$. A dispatch with $q + \beta_1 p > L$ is clearly not possible.



Letting $\Psi(q, p)$ denote the probability that we are dispatched at a point on our offer curve below (q, p) (as in [4]), we have for $q + \beta_1 p < L$

$$\begin{aligned}\frac{\partial \Psi}{\partial q}(q, p) &= f(q + (\beta_1 + \beta_2)p) \\ \frac{\partial \Psi}{\partial p}(q, p) &= f(q + (\beta_1 + \beta_2)p)(\beta_1 + \beta_2)\end{aligned}$$

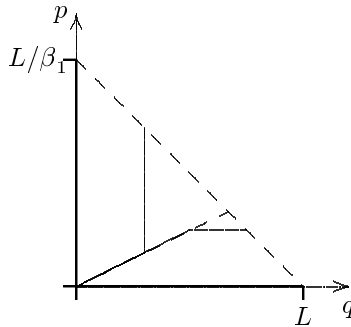
The expected revenue generated by an offer curve T with endpoint (q_T, p_T) is thus

$$\begin{aligned}\rho(T) = & \int_T qp f(q + (\beta_1 + \beta_2)p)(dq + (\beta_1 + \beta_2)dp) \\ & + \int_{q_T + (\beta_1 + \beta_2)p_T}^{\infty} (q_T p_T + q_f(\beta_2^{-1}(x - L) - p_T)) f(x) dx\end{aligned}\tag{1}$$

The optimum curve T may be determined as follows. Fix the endpoint (q_T, p_T) , and consider variations to the rest of T . Since the second term in (1) is now constant, the arguments of [4] show that the optimum curve must at each point either be horizontal or vertical, or satisfy the equation $Z = 0$, where

$$Z = p(\partial \Psi / \partial p) - q(\partial \Psi / \partial q) = ((\beta_1 + \beta_2)p - q)f(q + (\beta_1 + \beta_2)p).$$

>From this it is not hard to see that the optimum T must resemble one of the two solid curves in the diagram below.



(Note the role being played here by the requirement that the offer curve be monotone increasing in both p and q . If it were not for this constraint, the solution would be to choose T to be the curve $q = (\beta_1 + \beta_2)p$, then discontinuously jump to whatever endpoint (q_T, p_T) would optimize the second term of (1).)

Assuming this form for T , it is possible to write the first integral in (1) as a function of (q_T, p_T) . It then remains only to perform a one-dimensional optimization over (q_T, p_T) to determine the optimal T . The particular point (q_T, p_T) which turns out to be optimal will, in general, depend on f , i.e. on the demand distribution, as well as on q_f .

The effect of the FTR on this offer curve can easily be seen. The effect of q_f on $\rho(T)$ is to contribute the term

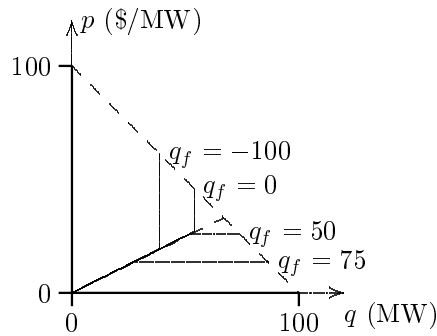
$$q_f \int_{L+\beta_2 p_T}^{\infty} \left(\frac{x-L}{\beta_2} - p_T \right) f(x) dx$$

which for $q_f > 0$ is a decreasing function of p_T . (Its derivative with respect to p_T is $-q_f P(D \geq L + \beta_2 p_T)$.) The optimum curve when $q_f > 0$ will therefore have smaller p_T than when $q_f = 0$.

In other words, our natural tendency to offer aggressively (withholding supply) will be reduced in the presence of the FTR. Observe, too, that this has the effects of reducing the expected price paid by consumers at node 2, and increasing the probability that the line constraint will become active.

If our FTR went in the other direction (from node 2 to node 1) it would instead lead to a more aggressive strategy. This can be seen by considering negative values of q_f .

For a numerical example, suppose that $L = 100\text{MW}$ and $\beta_1 = \beta_2 = 1\text{MW}^2/\$$, while the demand D has a normal distribution with mean 150MW and standard deviation 20MW . Optimal offer curves for several values of q_f are depicted below. When $q_f = 100\text{MW}$ (i.e. the full capacity of the line) the optimal curve has $p_T = \$0.30/\text{MW}$, which makes it hardly distinguishable from the q axis. In this case our strategy has been almost reduced to the competitive one, which is to offer at zero price.

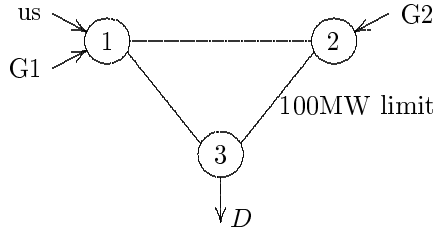


An interesting variant of this problem can be obtained by removing the competitive fringe at node 1. The dashed line is then replaced by a vertical one,

so the monotonicity constraint allows a jump discontinuity at the endpoint of T . It turns out that in this case, the FTR does not affect our behaviour. A similar problem (though without the full supply curve) is worked out in [11].

Example 8 *FTR exacerbating the effects of market power.*

Consider a three-node grid in which “we” are the only generator large enough to influence prices.



A competitive fringe consisting of many smaller generators provides additional supply “G1” and “G2”; which is offered to the market via fixed aggregated offer curves. For simplicity, assume that these take the forms $q = \beta_1 p$ and $q = \beta_2 p$ at nodes 1 and 2 respectively. Demand D is located at node 3 and is random, with a probability density function f . There are no line losses, and no line capacity constraints other than the 100MW limit on the line between nodes 2 and 3. The three lines have equal admittances. (The significance of this last point is that $1/3$ of any power injected at node 1, and $2/3$ of any power injected at node 2, must flow via the limited-capacity line to reach the load.) Suppose that we can generate power at no marginal cost, and that we own a balanced FTR for q_f megawatts from node 1 to node 3.

What supply curve should we offer, and how is this affected by the presence of the FTR? An analysis of the dispatch problem shows that the nodal prices p_1 , p_2 , and p_3 will always satisfy $p_3 - p_1 = p_1 - p_2$. Thus, for a given value of p_2 , increasing p_1 will also increase the difference $p_3 - p_1$ and hence the revenue from the FTR. This suggests that the price-maker’s usual incentive to force up the local price by withholding supply will be reinforced by the presence of the FTR. The following analysis shows that this is indeed the case.

Consider a point (q, p) through which our supply curve may pass. It may be that we are dispatched at the point (q, p) without constraining the line between nodes 2 and 3; this is possible if and only if

$$\frac{1}{3}(q + \beta_1 p) + \frac{2}{3}\beta_2 p < 100$$

since in this case the price will be p at every node. The level of load which achieves this is $D = q + (\beta_1 + \beta_2)p$.

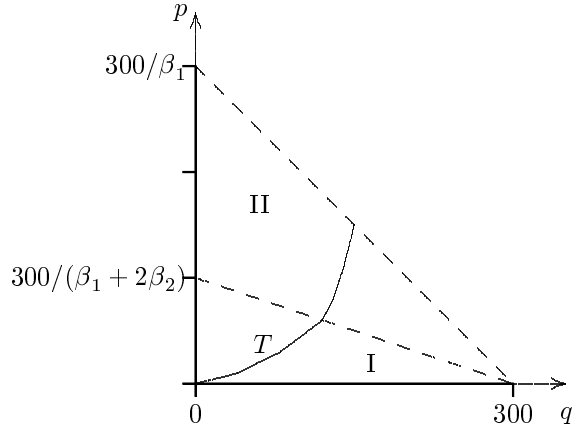
Now consider a point (q, p) with $q + (\beta_1 + 2\beta_2)p > 300$. It may still be possible to be dispatched at such a point, but only in a situation where the line

between nodes 2 and 3 is at capacity, and the nodal prices are unequal. This requires

$$\begin{aligned}\frac{1}{3}(q + \beta_1 p_1) + \frac{2}{3}\beta_2 p_2 &= 100 \\ q + \beta_1 p_1 + \beta_2 p_2 &= D \\ p_1 &= p \\ p_3 - p_1 &= p_1 - p_2.\end{aligned}$$

Solving, we find that $D = 150 + (q + \beta_1 p)/2$ and $p_2 = (300 - q - \beta_1 p)/2\beta_2$ (hence $p_3 - p_1 = p_1 - p_2 = (q + (\beta_1 + 2\beta_2)p - 300)/2\beta_2$). Note that this is positive. (In this model, every possible dispatch has $p_1 \geq p_2$ and so our revenue from the FTR will never be negative.) Such a dispatch is possible for any (q, p) with $q + \beta_1 p \leq 300$. For $q + \beta_1 p > 300$, the above equations are invalid, as they give $p_2 < 0$, and in that case the generation at node 2 should be 0 rather than $\beta_2 p_2$.

In fact, it is never possible for us to be dispatched at a point (q, p) with $q + \beta_1 p > 300$, since in that case the generation at node 1 alone will already be enough to violate the line capacity constraint between nodes 2 and 3.



Our offer curve T must therefore pass through two distinct regions, labelled I and II on the diagram. In region II, the line between nodes 2 and 3 will be at capacity; in region I, it will not.

Following [4], let $\Psi(q, p)$ denote the probability that we are dispatched at a point below (q, p) on our offer curve. (Here (q, p) is assumed to be a point on that curve.) The above remarks imply that for (q, p) in region I,

$$\begin{aligned}\frac{\partial \Psi}{\partial q}(q, p) &= f(q + (\beta_1 + \beta_2)p) \\ \frac{\partial \Psi}{\partial p}(q, p) &= f(q + (\beta_1 + \beta_2)p)(\beta_1 + \beta_2),\end{aligned}$$

while for (q, p) in region II,

$$\begin{aligned}\frac{\partial \Psi}{\partial q}(q, p) &= f(150 + (q + \beta_1 p)/2)/2 \\ \frac{\partial \Psi}{\partial p}(q, p) &= f(150 + (q + \beta_1 p)/2)\beta_1/2.\end{aligned}$$

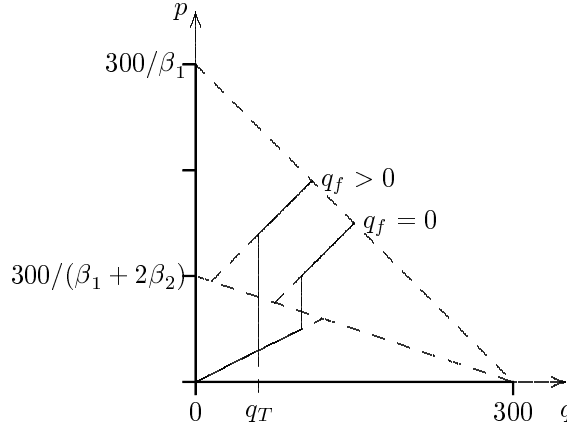
If we are dispatched at (q, p) , our total revenue is

$$\begin{aligned}R(q, p) &= qp_1 + q_f(p_3 - p_1) \\ &= \begin{cases} qp & \text{in region I} \\ qp + q_f(q + (\beta_1 + 2\beta_2)p - 300)/2\beta_2 & \text{in region II} \end{cases}\end{aligned}$$

According to [4], the optimal offer curve must at each point either be horizontal or vertical, or satisfy the equation $Z = 0$, where

$$\begin{aligned}Z &= \frac{\partial R}{\partial q} \frac{\partial \Psi}{\partial p} - \frac{\partial R}{\partial p} \frac{\partial \Psi}{\partial q} \\ &= \begin{cases} ((\beta_1 + \beta_2)p - q)f(q + (\beta_1 + \beta_2)p) & \text{in region I} \\ (\beta_1 p - q - q_f)\frac{1}{2}f(150 + (q + \beta_1 p)/2) & \text{in region II} \end{cases}\end{aligned}$$

The $Z = 0$ contour is thus $q = (\beta_1 + \beta_2)p$ in region I and $q = \beta_1 p - q_f$ in region II, independently of f . From this, it is not hard to see that the best offer curve must resemble the upper curve in the diagram below. (The value of q_T will in general depend on f .)



For comparison, the lower curve in the diagram shows the offer curve we would submit if we did not own the FTR. Ownership of the FTR thus requires more aggressive strategic offering (with respect to both quantity and price) on our part. As a result of this, the line capacity constraint between nodes 2 and 3 is more likely to come into play, and the expected price p_3 paid by consumers at node 3 is increased.

4 Allocating Rights by Auction

In this section we consider the design of the auction mechanism by which the ISO creates FTRs and distributes them to market participants.

Note that there are many more possible types of FTRs (e.g. a balanced FTR between any pair of nodes in the network) than are likely to be needed in practice. The ISO must thus make two decisions: (i) which FTRs should exist; and (ii) who should own them. In an auction framework, both decisions are driven by the bids received, subject only to the requirement that all the FTRs created must be simultaneously feasible in the sense discussed in section 2.

This line of thinking leads directly to an auction of the following kind. Suppose A bids are received, with bid α ($\alpha = 1, \dots, A$) offering F_α dollars in exchange for an FTR contract described by a vector $h(\alpha)$ (i.e. one which pays $\sum_i \pi_i h_i(\alpha)$). Then the auction is cleared by accepting a fraction r_α of each bid α in such a way as to maximize the resulting revenue to the ISO. Formally, the auctioneer solves:

$$\begin{array}{ll} \text{AP:} & \text{maximize} \quad \sum_{\alpha} F_{\alpha} r_{\alpha} \\ & \text{subject to} \quad \begin{array}{ll} g_i(y) - z_i &= \sum_{\alpha} r_{\alpha} h_i(\alpha), \quad i = 1, 2, \dots, n \\ r_{\alpha} &\in [0, 1] \quad \alpha = 1, 2, \dots, A \\ z_i &\geq 0, \quad i = 1, 2, \dots, n \\ y &\in U, \end{array} \end{array}$$

where g_i , U are as in section 2. Note that this is a linear program, and therefore relatively easy to solve. The prices actually paid by the bidders are determined from the dual problem. This type of auction is discussed further by Hogan in [10].

However, this auction design may fail to capture all of the revenue opportunities available to the ISO when there are bidders who have market power in the electricity market. To see why, we must consider what an FTR is worth to such a bidder.

Suppose a large generator is such that when it chooses an offer curve T , the resulting nodal price at node i has expectation $\bar{\pi}_i(T)$. (Cf. the examples in the previous section.) To this generator, the value of a fraction r of an FTR that pays $\sum_i h_i \pi_i$ is

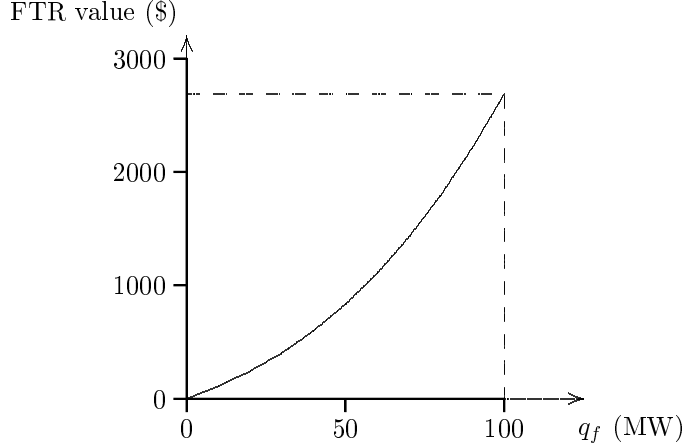
$$F(r) = \max_T \left(\rho(T) + r \sum_i h_i \bar{\pi}_i(T) \right) - \max_T \rho(T),$$

where $\rho(T)$ represents the value generated by all activities other than this fraction of this FTR (including energy trading, other FTRs, and perhaps a fixed further fraction r_0 of this same FTR). If $\max_T \rho(T)$ is attained for $T = T^*$, then we have

$$F(r) \geq \rho(T^*) + r \sum_i h_i \bar{\pi}_i(T^*) - \rho(T^*) = r \sum_i h_i \bar{\pi}_i(T^*)$$

>From this it follows that F is a convex function, and linear only when the π_i do not depend on T .

As an example, consider the large upstream generator in (the numerical specialisation of) Example 7. If this generator owns q_f megawatts of the FTR over the line, then its expected total revenue may be found from equation (1).



Note that the derivative of this function is at every point equal to the “investment” value of a unit FTR, i.e. $E[\pi_2(T^*) - \pi_1(T^*)]$, where T^* is the optimal offer curve for the given value of q_f .

The market for FTRs in the presence of such a strategic bidder is thus not dissimilar to the market for a public company’s shares in the presence of a strategic buyer to whom a large block of shares would have an enhanced value associated with control. To the strategic bidder, a small additional holding of FTRs has the same value as it would to anyone else, but a higher price can be paid for a large block. (One difference is that a shareholder’s control over a public company increases discontinuously at a shareholding level of 50%, while the value-enhancement possible with FTRs increases continuously with the quantity held.) This analogy was first made by Joskow and Tirole in [11].

In a simple auction of the type discussed above, the strategic bidder faces a difficult problem. If it bids F_α for an FTR $h(\alpha)$, the bid will be scaled back by some factor r_α – which is unknown at the time the bid is made – so that the actual FTR purchased is $r_\alpha h(\alpha)$, for an amount $r_\alpha F_\alpha$. The bidder will therefore not want to set F_α to be the full amount that it perceives $h(\alpha)$ to be worth, since this is likely to result in it overpaying for the FTR eventually received. This will be especially so in a complex network, where the bid must compete against other participants’ bids for FTRs between many other pairs of nodes in order to maintain simultaneous feasibility overall, and the value of r_α is thus especially uncertain in advance.

A possible modification of the auction which would accomodate the strategic bidders (and therefore enhance the ISO’s likely revenue) would be to allow some

bids to be conditional on others being fully accepted. A strategic bidder could then offer a higher price for further FTRs bought after a significant holding had already been acquired. However, this would make the auctioneer's problem more complex – instead of a linear program, it would become a mixed-integer linear program.

In the light of the previous section, it could be argued that the fundamental objective of the auction – to maximize the ISO's revenue – should be sacrificed in an effort to prevent strategic players from obtaining large blocks of FTRs, and the consequent effects on the energy market. Counter-arguments would be: (i) Awarding large blocks of FTRs to price-makers need not be a bad thing; it can be very beneficial, as Example 7 shows; (ii) even if the auction were designed so as to thwart the strategic bidders, they might still be able to get the FTRs they wanted during post-auction secondary trading. The additional amount strategic bidders are prepared to pay for FTRs would then be captured by intermediaries, instead of by the ISO.

5 Grid Investment

In this section we investigate some of the advantages that FTRs provide in supporting investment in transmission assets. It is well-known (see e.g. [13]) that electricity transmission networks support Braess-type paradoxes, in which the addition of transmission capacity (or admittance) results in a loss in producer and consumer surplus. If the addition of this capacity were to benefit a single market participant at the expense of others then it should be discouraged in some way. We show by example how FTRs can be used to provide a disincentive for detrimental network expansions.

It is important to be precise here about what is meant by a detrimental network expansion. Recall the dispatch problem

$$\begin{array}{ll} \text{P:} & \text{minimize} \quad \sum_i \sum_{j \in O(i)} c_j x_j \\ & \text{subject to} \quad g_i(f) + \sum_{j \in O(i)} x_j - z_i = d_i, \quad i = 1, 2, \dots, n \\ & \quad \quad \quad z_i \geq 0, \quad i = 1, 2, \dots, n \\ & \quad \quad \quad x \in X \\ & \quad \quad \quad f \in U \end{array}$$

When the offer price c_j is the marginal cost of generation the objective function of problem P is the cost of meeting an inelastic demand. If one assumes for each node i that all marginal demand below the level d_i is worth a very high fixed price \bar{c} to the consumer, and worth zero above d_i then minimizing the cost of dispatch by solving P is equivalent to maximizing the consumer and producer benefit defined by

$$B = \sum_i (\bar{c} d_i - \sum_{j \in O(i)} c_j x_j).$$

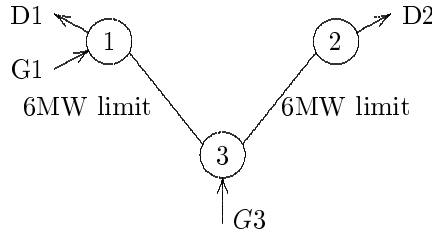
Given an optimal dispatch and corresponding shadow prices π we can rewrite the consumer and producer benefit as

$$B = \sum_i (\bar{c} - \pi_i) d_i + \sum_i \pi_i (d_i - \sum_{j \in O(i)} x_j) + \sum_i \sum_{j \in O(i)} (\pi_i - c_j) x_j.$$

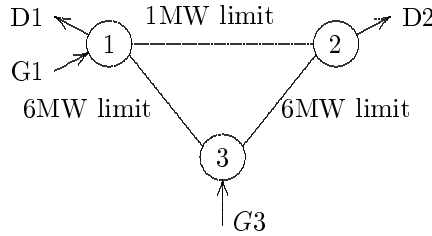
The three terms in this expression are the consumer surplus, the constraint rentals, and the producer surplus. Thus, given any optimal dispatch the profit is split amongst purchasers, the ISO, and generators according to the formula above. We say an expansion (or contraction) of the transmission network is *detrimental* if it decreases the consumer and producer benefit. Note that this is not the same as decreasing the total consumer and producer surplus since the consumer and producer benefit includes the constraint rentals.

Example 9 *Detrimental network expansions and contractions*

The following three node example is based on that of Bushnell and Stoft [6].



Before expansion the transmission lines have 6MW capacities and the nodes have demands of $D1=20\text{MW}$ and $D2=2\text{MW}$. The generator $G3$ offers power in unlimited quantities at \$10 per MWhr and $G1$ offers unlimited quantities at \$15 per MWhr. The optimal dispatch sends 6MW to node 1 from node 3 and 2MW from node 3 to node 2. The nodal prices are \$15, \$10, and \$10 for nodes 1, 2, and 3 respectively. If we let $\bar{c} = 20$, then the consumer surplus is \$120, the producer surplus is \$0, and the ISO surplus = \$30.



As an example of a detrimental expansion of the network consider the same transmission system after a 1MW line is added between node 1 and node 2, where we assume that all lines have the same admittance. The optimal dispatch is now $f_{31} = 4$, $f_{32} = 3$, and $f_{21} = 1$, and the nodal prices are $\pi_1 = \$15$, $\pi_2 = \$5$, and $\pi_3 = \$10$. Still assuming $\bar{c} = 20$, we obtain a consumer surplus of \$130, a producer surplus of \$0, and an ISO surplus of \$15, giving a decrease in consumer and producer benefit from \$150 to \$145. This is an interesting example since the consumers would prefer the expansion, the generators would be indifferent, and the ISO would lose from the expansion.

The above example is also of interest since it illustrates that removing the 1MW line from 1 to 2 would increase the ISO's line rentals and yet make no other market participants better off - indeed the consumers at node 2 would be worse off. The generation cost decreases to \$295, but generators continue to make zero profit. The removal of this line would not be detrimental since it would result in an improvement in the consumer and producer benefit.

As discussed by Bushnell and Stoft [5] it is possible to discourage detrimental investments using FTRs. Consider the set of FTRs

$$\{h(\alpha)\} = \{[6 \ 0 \ -6]^\top, [0 \ 2 \ -2]^\top\}$$

for the network before expansion. These match the dispatch in the network. It is easy to show that this set of FTRs is no longer simultaneously feasible for the network with the additional line. Consider now adding the FTR $\{[-1 \ 0 \ 1]^\top\}$ to $\{h(\alpha)\}$. This now gives a simultaneously feasible set of FTRs. The coupon payment for this FTR from the optimal dispatch after the network expansion is -\$5, which exactly matches the loss in consumer and producer benefit (from \$150 to \$145). Thus the (negative) coupon payment from the FTR allocated to the investor in the detrimental line will cover the losses experienced by the market participants.

This example illustrates a general result due to Bushnell and Stoft [5] for networks with quadratic losses, and proved below for general convex dispatch problems. This states that if FTRs match dispatch then any addition of transmission capacity that results in a loss in producer and consumer benefit will be assigned an FTR that transfers at least that loss to the holder of the FTR.

Formally, suppose that (x^*, f^*, z^*) solves P for a network with $z \geq 0$, $x \in X$, $f \in U$, under a given demand d . Suppose $\sum_{\alpha} r_{\alpha}^* h_i^*(\alpha)$ is a set of FTRs that are held by market participants that match (x^*, f^*, z^*) . That is

$$\sum_{\alpha} r_{\alpha}^* h_i^*(\alpha) = d_i - \sum_{j \in O(i)} x_j^*$$

Now suppose an agent with no other interest in the network were to build a new set of lines so that now $f \in U'$, and by virtue of this investment it is granted an FTR $r_{A+1}^* h_i^*(A+1)$ for the lifetime of the assets. This must be simultaneously feasible with existing FTRs. Suppose (x', f', z') solves P for a network with $z \geq 0$, $x \in X$, $f \in U'$, under a given demand d , and yields nodal prices π' .

Theorem 10 Suppose for some $\Delta > 0$, $\sum_i \sum_{j \in O(i)} c_j x'_j = \sum_i \sum_{j \in O(i)} c_j x_j^* + \Delta$. Then any new FTR allocated to the builder of the new line will have a value no more than $-\Delta$.

Proof. Recall that there is a set of optimal Lagrange multipliers π' such that (x', f', z') minimizes the Lagrangian

$$\mathcal{L}(x, f, z) = \sum_i \sum_{j \in O(i)} c_j x'_j + \sum_i \pi'_i (d_i - g_i(f')) - \sum_{j \in O(i)} x'_j + z'_i$$

over $z' \geq 0$, $x' \in X$, $f \in U'$. Observe that this entails that $\sum_i \sum_{j \in O(i)} c_j x'_j - \sum_i \pi'_i \sum_{j \in O(i)} x'_j$ is minimized over all $x' \in X$, and so since $x^* \in X$,

$$\sum_i \sum_{j \in O(i)} c_j x'_j - \sum_i \pi'_i \sum_{j \in O(i)} x'_j \leq \sum_i \sum_{j \in O(i)} c_j x_j^* - \sum_i \pi'_i \sum_{j \in O(i)} x_j^*,$$

giving

$$\sum_i \pi'_i \sum_{j \in O(i)} x_j^* - \sum_i \pi'_i \sum_{j \in O(i)} x'_j \leq -\Delta. \quad (2)$$

Now by assumption

$$\sum_{\alpha} r_{\alpha}^* h_i^*(\alpha) = d_i - \sum_{j \in O(i)} x_j^*$$

so 2 implies that

$$\sum_i \pi'_i (d_i - \sum_{\alpha} r_{\alpha}^* h_i^*(\alpha)) - \sum_i \pi'_i \sum_{j \in O(i)} x'_j \leq -\Delta. \quad (3)$$

Now observe that the FTR auction after the expansion give a rights allocation r'_{α} satisfying

$$\sum_{\alpha} r'_{\alpha} h'_i(\alpha) = \sum_{\alpha} r_{\alpha}^* h_i^*(\alpha) + r_{A+1}^* h_i^*(A+1),$$

and Theorem 5 implies that

$$\sum_i \pi'_i \sum_{\alpha} r'_{\alpha} h'_i(\alpha) \leq \sum_i \pi'_i d_i - \sum_i \pi'_i \sum_{j \in O(i)} x'_j.$$

giving

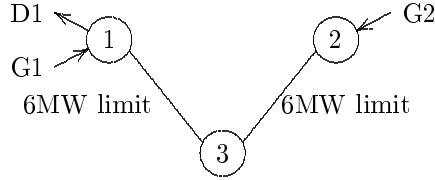
$$\begin{aligned} \sum_i \pi'_i r_{A+1}^* h_i^*(A+1) &= \sum_i \pi'_i \sum_{\alpha} r'_{\alpha} h'_i(\alpha) - \sum_i \pi'_i \sum_{\alpha} r_{\alpha}^* h_i^*(\alpha) \\ &\leq \sum_i \pi'_i d_i - \sum_i \pi'_i \sum_{j \in O(i)} x'_j - \sum_i \pi'_i \sum_{\alpha} r_{\alpha}^* h_i^*(\alpha) \\ &\leq -\Delta. \end{aligned}$$

■

Theorem 10 provides some assurance that detrimental investments in transmission lines can be discouraged. Of course the theorem is conditional on the premise that the FTRs held before expansion match the dispatch, and it is not obvious what the extent of the FTR disincentive will be in the case when the dispatch deviates from this. A second remark worth making is that the FTR allocated to the investor represents an obligation rather than an option. We can illustrate why this is necessary by considering a final example.

Example 11 *FTRs need to be obligations*

Consider the following three-node transmission network.



Before expansion the transmission lines have 6MW capacities and node 1 has a demand of $D1=5\text{MW}$. The generator $G1$ offers unlimited quantities at \$2 per MWhr, and $G2$ offers unlimited quantities at \$1 per MWhr. The optimal dispatch sends 5MW to node 1 from node 2 via node 3. The nodal prices are \$1 at every node.

Now suppose that $G1$ invests in a transmission line of 2 MW between nodes 1 and 2, where as before all lines have equal admittance. After the expansion the shadow price at node 1 is \$2. Prior to investment the maximum possible simultaneously feasible FTR between nodes 2 and 1 is $\begin{bmatrix} 6 & -6 & 0 \end{bmatrix}^\top$. After investment the maximum possible FTR between nodes 2 and 1 is $\begin{bmatrix} 3 & -3 & 0 \end{bmatrix}^\top$. The difference between these FTRs is a new FTR of $\begin{bmatrix} -3 & 3 & 0 \end{bmatrix}^\top$ that is offered to $G1$. The coupon payment for this FTR is -\$3. If $G1$ declines this (as it should if given the option) then the consumer and producer benefit decreases, while $G1$ is dispatched at \$2 and so is better off.

6 Conclusion

As instruments for hedging nodal price differences, FTRs provide a valuable tool for market participants to use. This paper has illuminated some of the incentives that these instruments provide to market participants to enhance their profit. Like contracts for differences, FTRs may mitigate the market power of those

holding them. However they may also enhance this market power (just as a generator buying a contract for differences will have an incentive to set high spot prices). The increasing marginal value of FTRs poses problems for the auction mechanism, which implicitly treats this as constant for any level of the FTR held. To maximize the auction revenue, the auction might need to be run as a mixed integer program.

Revenue adequacy is a key property desired of FTRs. The simultaneous feasibility condition will guarantee this as long as the network is unaffected by outages, and there are no negative nodal prices. It should be noted that the presence of FTRs might serve as an incentive to produce negative prices, so some methodology of dealing with these needs to be in place, in case they become a lot more frequent.

FTRs play a key role in allocating (at least part of) the costs and benefits of new investment to the investors. It is important that this allocation procedure be well understood. In particular investors should be obliged to accept FTRs in the event that their investment is detrimental.

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