On Load Shedding and Transmission Grid Security

A.B. Philpott and G.R. Everett
Department of Engineering Science
University of Auckland
and
Norske Skog Limited
a.philpott@auckland.ac.nz

Abstract

The New Zealand transmission grid operator Transpower uses special constraints in the SPD dispatch software to ensure that in the event of a single transmission line failure, remaining circuits are not over-loaded. We discuss a mechanism by which these constraints might be able to be relaxed by making use of interruptible load.

1 Introduction

The New Zealand grid operator Transpower is responsible for the real-time scheduling and coordination of electricity generation and transmission. A detailed description of Transpower's responsibilities is available in [4, Appendix C], and we restrict ourselves here to a brief summary. The scheduling of dispatch is carried out by Transpower using a linear program called the Scheduling Pricing and Dispatch (SPD) model. SPD constructs a dispatch and transmission schedule to meet a forecast load at least cost, as measured by generator offers. When scheduling the dispatch, SPD includes constraints on groups of circuits that are designed to guarantee that the scheduled dispatch allows for an $N-1$ level of security. This means that the system should be robust to the failure of any single component, here called a *contingency*.

The constraints to be added to SPD are determined offline by proprietary Transpower software called ConAmp and Overload. ConAmp models the change in temperature of a circuit when it is suddenly required to carry more current. When conductors become hot they expand and start to sag. The agreed security requirement states that the temperature of the circuit should remain below a maximum threshold level (determined by its maximum amount of sag) for at least 15 minutes after the sudden increase. This gives operators sufficient time to take some restorative action, before the line sags too much. The algorithm in ConAmp is based on formulae originally due to Latta [2], and is outlined in [1] and [3]. The program can be used to compute an approximate bound on the current in the conductor after the contingency in terms of a linear decreasing function of the current in the conductor before the contingency. This forms the basis of the security constraint.

Adding constraints to SPD will generally increase the optimal objective function value, and can result in higher electricity prices and constraint rentals (although these effects may not be uniform across the grid). Prices and rentals provide signals for investment to improve the economic outcomes of the agents making the investment. Typically these investments are of the form of distributed generation or investments in grid capacity.

An alternative mechanism for alleviating the effects of security constraints is to use interruptible load at grid exit points for instantaneous demand reduction. (Generation reserve at injection nodes can play a similar role.) The availability of interruptible load that is able to immediately be removed in response to a direct signal (called an *inter trip*) after a contingent event allows the standard security constraints to be relaxed by adding a simple term to the right-hand side. In this paper we study this mechanism, and investigate how it might be used to amend the standard security constraints in SPD.

This paper is primarily a theoretical exercise - we do not discuss how inter-trip arrangements between Transpower and interruptible load bidders might be negotiated or physically implemented. We assume throughout that these can be put in place, and can be operated in a reliable fashion. The main contribution of the paper is to show that (possibly accidental) removal of some part of the interruptible load, even after generation has been dispatched, can be incorporated by using appropriate distribution factors. The required level of security is provided, even when the dispatcher does not know the level of interruptible load removed.

The next section describes the basic inter-trip model and then relates this to standard security constraints. It is also demonstrated that the inter-trip model is valid if some (unknown) amount of interruptible load is removed. In section 3 we conclude with an example.

2 Inter-trip model

Let the risk be defined by contingent line k, that contains pre-contingency current I_k, and we wish to protect a different line, j say, from overloading if line k should fail. Consider the pre-contingency current flow I_j in line j. We define:

$$\begin{aligned}
U &= \text{Interruptible current available at bus } i \\
\alpha_{kj} &= \text{distribution factor for current from conductor } k \text{ to conductor } j \\
\beta_{ij} &= \text{distribution factor for current from bus } i \text{ to conductor } j \text{ when } k \text{ is} \\
&\quad\ \text{disconnected} \\
\gamma_{ij} &= \text{distribution factor for current from bus } i \text{ to conductor } j \text{ while } k \text{ is} \\
&\quad\ \text{still connected} \\
\gamma_{ik} &= \text{distribution factor for current from bus } i \text{ to conductor } k \text{ while } k \text{ is} \\
&\quad\ \text{still connected}
\end{aligned}$$

It is useful here to interpret these distribution factors as they pertain to the loss of line k. The factor α_{kj} gives the proportion of current in line k that is diverted onto line j when k fails. The factor γ_{ij} is a pre-contingency sensitivity factor for line j (computed before line k fails) that gives the proportion of current drawn at bus i that is carried by the line j. Similarly γ_{ik} is the pre-contingency sensitivity factor for line k. (We do not use the γ factors until the next section.) Finally the factor β_{ij} is the post-contingency sensitivity factor for line j that gives the proportion of current drawn at bus i that is carried by the line j *after* line k fails. The sign convention here is that β_{ij}, γ_{ij}, and γ_{ik} measure the increase in current in conductors j and k for a unit increase in current drawn at i. A methodology for computing β_{ij}, γ_{ij}, and γ_{ik} is described in section 2.

In this paper we assume a unit power factor, and for convenience we assume a constant voltage throughout the system modelled. The latter assumption means that distribution factors can be found by computing sensitivity factors from a DC-Load Flow model (see [5, Chapter 11]). The distribution factors obtained in this way give changes in *power* flow in the lines j and k, that translate directly to changes in current flow as long as the interruptible load is expressed as an interruptible current at the common voltage. If the lines j and k operate at different voltages from the load bus i, or each other, then the distribution factors need to be scaled by the corresponding voltages to give changes in *current* in the lines j and k that result from changes in current at i or the loss of line k.

The pre-contingency current flow I_j in conductor j, and the post-contingency flow I'_j in conductor j together satisfy

$$aI_j + I'_j \leq c, \tag{1}$$

where a and c are constants for the line that come from ConAmp. We remark that (1) is the key inequality that must be satisfied by pre-contingency and post-contingency currents to ensure that the current in the line does not meet its threshold current within 15 minutes after the contingency. The post-contingency current depends on how the system reacts to the contingency. Our analysis in this paper is directed at examining the effects of tripping interruptible load on the constraint (1).

The standard approach now adopted in SPD is to allow the current on the failed line to travel by alternative links. If conductor k should fail then conductor j will carry a post-contingency current of

$$I'_j = I_j + \alpha_{kj} I_k,$$

leading to the standard security constraint

$$(a+1)I_j + \alpha_{kj} I_k \leq c. \tag{2}$$

Now suppose the interruptible load is shed immediately resulting in a reduction in current of U at bus i. Then the post-contingency current in conductor j will be

$$I'_j = I_j + \alpha_{kj} I_k - \beta_{ij} U. \tag{3}$$

This gives the following version of (1):

$$aI_j + I_j + (\alpha_{kj} I_k - \beta_{ij} U) \leq c.$$

Rearranging gives

$$(a+1)I_j + (\alpha_{kj} I_k - \beta_{ij} U) \leq c,$$

resulting in the adjusted security constraint

$$(a+1)I_j + \alpha_{kj} I_k \leq c + \beta_{ij} U. \tag{4}$$

The right-hand side of the standard security constraint (2) is increased by the distribution factor multiplied by the interruptible current. If I_j and I_k satisfy (4) then it is easy to see by substituting (3) into (4) that

$$aI_j + I'_j \leq c.$$

2.1 Interruptible load not available

Now suppose that some amount ρU (where $\rho \in [0,1]$) of the interruptible current U is for some reason not available at bus i, after U has been dispatched. This means that $(1-\rho)U$ is now the interruptible current available at bus i. Suppose the dispatcher believes that the currents in j and k are I_j and I_k. In reality, because some load has already been shed, these currents are $\bar{I}_j = I_j - \gamma_{ij}\rho U$, and $\bar{I}_k = I_k - \gamma_{ik}\rho U$. Assume for the moment that the temperature of conductor j has reached its steady state while carrying current $\bar{I}_j$. From ConAmp, we require for 15-minute security that

$$a\bar{I}_j + \bar{I}'_j \leq c,$$

where $\bar{I}'_j$ is the post-contingency current in circuit j. However the dispatcher does not know $\bar{I}_j$ and $\bar{I}_k$. Nevertheless, suppose the dispatcher imposes the following security constraint on the dispatch flows

$$(a+1)I_j + \alpha_{kj}I_k \leq c + \mu U, \tag{5}$$

where

$$\mu = (1-\rho)\beta_{ij} + \rho((a+1)\gamma_{ij} + \alpha_{kj}\gamma_{ik}).$$

Then it follows that

$$(a+1)I_j + \alpha_{kj}I_k \leq c + \beta_{ij}(1-\rho)U + (a\gamma_{ij} + \gamma_{ij} + \alpha_{kj}\gamma_{ik})\rho U,$$

so

$$a(I_j - \gamma_{ij}\rho U) + (I_j - \gamma_{ij}\rho U) + \alpha_{kj}(I_k - \gamma_{ik}\rho U) - \beta_{ij}(1-\rho)U \leq c. \tag{6}$$

Now $\bar{I}_j = I_j - \gamma_{ij}\rho U$, and $\bar{I}_k = I_k - \gamma_{ik}\rho U$ are the true pre-contingency currents because node i has shed current ρU after dispatch but before the contingency. So if k fails then $\alpha_{kj}(I_k - \gamma_{ik}\rho U)$ is transferred to circuit j and $(1-\rho)U$ is shed to give the actual post-contingency current of

$$\bar{I}'_j = (I_j - \gamma_{ij}\rho U) + \alpha_{kj}(I_k - \gamma_{ik}\rho U) - \beta_{ij}(1-\rho)U. \tag{7}$$

It follows by substituting $\bar{I}'_j$ given by (7) and $\bar{I}_j = I_j - \gamma_{ij}\rho U$ into (6) that

$$a\bar{I}_j + \bar{I}'_j \leq c,$$

so the true (but unknown) currents satisfy the 15 minute security constraint determined by ConAmp.

Therefore to satisfy the 15 minute security constraint determined by ConAmp, a security constraint of the form (5) where

$$\mu = (1-\rho)\beta_{ij} + \rho((a+1)\gamma_{ij} + \alpha_{kj}\gamma_{ik}) \tag{8}$$

can be used to replace the original security constraint. The interpretation on μ is that it has two parts. The first term is the available interruptible load used as in the previous section. The second term gives the reduction in contingent action required because some load has been already shed. This produces two effects, a reduction in the original current $\gamma_{ij}\rho U$, as well as a reduction $\gamma_{ik}\rho U$ of the current in conductor k.

2.2 Conductor Temperature Effects

In the previous section we discussed how the dispatcher might compute μ for the case where some unknown quantity of interruptible load has been shed at node i prior to the contingency. The calculation of μ relied on the assumption that the temperature of conductor j had reached its steady state while carrying current $\bar{I}_j = I_j - \gamma_{ij}\rho U$. However, although the current in conductor j immediately drops to $\bar{I}_j = I_j - \gamma_{ij}\rho U$ after tripping ρU, the temperature of conductor j does not drop immediately. The true temperature will lie somewhere between its steady state level with current $\bar{I}_j$ and its steady state level with current I_j. In the worst case, where ρU of interruptible current U is shed from bus i immediately before the contingency occurs, the temperature will be as if the pre-contingency current were I_j.

In the worst case, to maintain the temperature below its sag limit for 15 minutes, the Latta formula would require $\bar{I}'_j$, the post-contingency current in circuit j to satisfy

$$aI_j + \bar{I}'_j \leq c,$$

the same constraint as in the previous section, but with I_j replacing $\bar{I}_j$. Now, suppose the dispatcher imposes the following security constraint on the dispatch flows

$$(a+1)I_j + \alpha_{kj}I_k \leq c + \mu U, \tag{9}$$

where

$$\mu = (1-\rho)\beta_{ij} + \rho(\gamma_{ij} + \alpha_{kj}\gamma_{ik}).$$

Then it follows that

$$(a+1)I_j + \alpha_{kj}I_k \leq c + \beta_{ij}(1-\rho)U + (\gamma_{ij} + \alpha_{kj}\gamma_{ik})\rho U,$$

so

$$aI_j + I_j - \gamma_{ij}\rho U + \alpha_{kj}(I_k - \gamma_{ik}\rho U) - \beta_{ij}(1-\rho)U \leq c. \tag{10}$$

Now $\bar{I}_j = I_j - \gamma_{ij}\rho U$, and $\bar{I}_k = I_k - \gamma_{ik}\rho U$ are the true pre-contingency currents because node i has shed current ρU after dispatch but before the contingency. So if k fails then $\alpha_{kj}(I_k - \gamma_{ik}\rho U)$ is transferred to circuit j and $(1-\rho)U$ is shed to give the actual post-contingency current of

$$\bar{I}'_j = (I_j - \gamma_{ij}\rho U) + \alpha_{kj}(I_k - \gamma_{ik}\rho U) - \beta_{ij}(1-\rho)U. \tag{11}$$

It follows by substituting $\bar{I}'_j$ given by (11) into (10) that

$$aI_j + \bar{I}'_j \leq c,$$

so the post contingency current will satisfy the 15 minute security constraint determined by ConAmp.

Therefore to satisfy the 15 minute security constraint determined by ConAmp, a security constraint of the form (9) where

$$\mu = (1-\rho)\beta_{ij} + \rho(\gamma_{ij} + \alpha_{kj}\gamma_{ik})$$

can be used to replace the original security constraint. The second term of μ now gives the reduction in contingent action required because some load has been already shed, which reduces the post contingent current that will be need to be accommodated in line j by the amount $\gamma_{ij}\rho U + \gamma_{ik}\rho U$.

2.3 Load shedding using an inter trip

Recall from the previous section that an equation constraint of the form

$$(a+1)I_j + \alpha_{kj} I_k \leq c + \mu U, \tag{12}$$

where

$$\mu = (1-\rho)\beta_{ij} + \rho(\gamma_{ij} + \alpha_{kj}\gamma_{ik})$$

will provide the correct level of 15 minute security for a failure of line k. If the operator correctly predicts that ρ will be 0, so all interruptible load is available, then μ can be chosen to be β_{ij}, and if the operator can anticipate that $\rho = 1$ (even though he dispatches assuming $\rho = 0$), then he chooses

$$\mu = \gamma_{ij} + \alpha_{kj}\gamma_{ik}.$$

Observe that in practice the system operator will not know the true value of ρ at the time when the dispatch is computed. Since $\rho \in [0,1]$, to ensure security under all possible values of ρ, the operator should choose

$$\mu = \min_{\rho \in [0,1]} (1-\rho)\beta_{ij} + \rho(\gamma_{ij} + \alpha_{kj}\gamma_{ik}),$$

so

$$\mu = \min\{\beta_{ij}, \gamma_{ij} + \alpha_{kj}\gamma_{ik}\}. \tag{13}$$

2.4 Discussion

On a superficial level, the above mechanism for load-shedding using an inter trip to alleviate constraints is very simple. However there is an important difference between tripping interruptible load before the contingency and tripping it after the contingency. The models that determine the current flow in each case have different sets of lines (giving different distribution factors). If we imagine the amount ρU of interruptible load shed being tripped at some random time t, then the ability of the interruptible load to assist with security is discontinuous as t moves across the time instant when the contingency occurs. It is this discontinuity that makes the modelling of this problem more complicated than it appears to be at first sight.

A second source of complication comes from the fact that t is random. If the system could guarantee that all the interruptible load at node i is available to be shed after the contingency, then this provides $\beta_{ij}U$ to alleviate the constraint. Since t is random, a guarantee of this amount of interruptible load to be shed after a contingency is not available. The model we have described above investigates how to provide security assuming a worst case situation regarding the availability of interruptible load. The worst value of t the system operator could encounter would occur just before a contingency, since the temperature of conductor j when the contingency (in k) occurs will still be high, even though the current has been reduced. To handle this, we assume the original current I_j in conductor j, which reduces the second term in the right-hand side of (8) by $\rho a \gamma_{ij}$. For any t, the worst case value of ρ will either be

1. when the interruptible load has not been shed at all, yet provides little relief to conductor j after the contingency because β_{ij} is small, or

2. when the interruptible load has all been shed, and the relief to conductor j after the contingency comes from the slightly lesser post-contingency current that j will now be required to carry.

Together these give the formula (13) for μ.

Finally, observe in some cases that μ will be zero, and no relaxation of the standard security constraint will be possible. For example, if the reference bus (discussed in section 3) is the same as node i then $\beta_{ij} = 0$. This will mean that any inter-trip of load at node i will have no effect on conductor j. A similar effect will occur if there is a unique chain of lines linking the reference bus to node i.

3 Inter-trip example

In this section we consider for the example network, the effect of protecting line j =2 against a failure in line k =1 when there is some interruptible load at node 4. The security constraint on the dispatch flows is

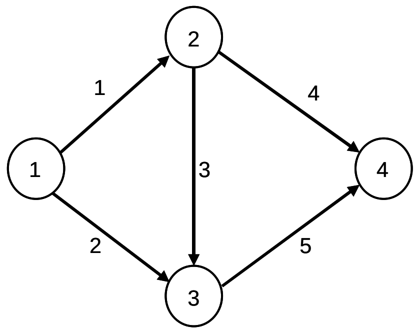

Figure 1: Four node example network

$$(a+1)I_j + \alpha_{kj} I_k \leq c + \mu U,$$

where

$$\mu = \min\{\beta_{ij}, \gamma_{ij} + \alpha_{kj}\gamma_{ik}\}.$$

Suppose $a = 0.31$ and recall that

$$\begin{aligned} i &= 4, \quad j = 2, \quad k = 1, \\ \gamma_{41} &= 0.5538, \quad \gamma_{42} = 0.4463, \quad \beta_{42} = \alpha_{12} = 1. \end{aligned}$$

Thus

$$\gamma_{42} + \alpha_{12}\gamma_{41} = ((0.4463) + (1.0)(0.5538)) = 1.0,$$

giving

$$\mu = \min\{\beta_{42}, \gamma_{42} + \alpha_{12}\gamma_{41}\} = \min\{1.0, 1.0\} = 1.$$

Taking the minimum gives an adjusted security constraint of

$$(1.31)I_2 + (1.0)I_1 \leq c + U,$$

where U is the maximum current that can be tripped at node 4.

To see how this constraint might affect the dispatch, consider a situation in which ConAmp gives the values $a = 0.31$, $c = 3000$. Thus for 15 minute security it is required that pre-contingency and post-contingency currents satisfy

$$(0.31)I_2 + I_2' \leq 3000.$$

This gives the standard security constraint

$$(1.31)I_2 + (1.0)I_1 \leq 3000.$$

As an illustrative example, suppose that the loads and injections are

$$P = \begin{bmatrix} 1000 \\ 25 \\ -375 \\ -650 \end{bmatrix}$$

giving

$$f^\top = \begin{bmatrix} 500 & 500 & 125 & 400 & 250 \end{bmatrix}.$$

Since

$$f_1 = 500 \quad f_2 = 500,$$

we have

$$I_1 = 500/(220\sqrt{3}) = 1312.16A \quad I_2 = 500/(220\sqrt{3}) = 1312.16A$$

giving

$$(1.31)I_2 + (1.0)I_1 = 3031.09,$$

which violates the standard security constraint.

Now suppose node 4 has 100 MW of interruptible load. This amounts to a current of

$$U = 100/(220\sqrt{3}) = 262.43\ A$$

This gives the adjusted security constraint

$$(1.31)I_2 + I_1 \leq c + U,$$

or

$$(1.31)I_2 + (1.0)I_1 \leq 3262.43, \tag{14}$$

that is now satisfied by the dispatch currents.

We know from the arguments above that if the dispatch currents satisfy (14) then I_2 and the true post-contingency current $\bar{I}_2'$ will always satisfy

$$(0.31)I_2 + \bar{I}_2' \leq 3000,$$

even if an unknown fraction of the interruptible load is shed, and so not available. To verify this for this example suppose that the 100 MW of interruptible load has been accounted for in the dispatch but is already tripped. The true power flows in the lines will be

$$\bar{f}_2 = 500 - 100\gamma_{42} = 500 - 44.63 = 455.37,$$

and

$$\bar{f}_1 = 500 - 100\gamma_{41} = 500 - 55.38 = 444.62,$$

so the true currents are

$$\bar{I}_1 = 444.62/(220\sqrt{3}) = 1166.8A \quad \bar{I}_2 = 455.37/(220\sqrt{3}) = 1195A.$$

Now after line 1 fails we obtain

$$\bar{I}_2' = 1195 + 1166.8.$$

However the worst-case temperature for line 2 (obtained when the 100 MW load is shed just prior to the contingency) corresponds to the (higher) current

$$I_2 = 500/(220\sqrt{3}) = 1312.16A.$$

So I_2 and $\bar{I}_2'$ satisfy

$$\begin{aligned}(0.31)I_2 + \bar{I}_2' &= (0.31)1312.2 + +1166.8 + 1195 \\ &= 2768 \\ &\leq 3000\end{aligned}$$

as required by ConAmp.

To check the other extreme, suppose that the 100 MW of interruptible load has been accounted for in the dispatch but is not tripped prior to the contingency. Then the true currents in the lines will be

$$\bar{I}_2 = 500/(220\sqrt{3}) = 1312.16A \quad \bar{I}_1 = 500/(220\sqrt{3}) = 1312.16A.$$

Now after line 1 fails, line 2 absorbs its current and the inter-trip is enacted to give

$$\bar{I}_2' = 1312.16 + (1)1312.16 - (1)262.43$$

where the second term comes from the extra current from the failed conductor 1, and the third term comes from the tripped load at node 4. Therefore

$$\begin{aligned}(0.31)\bar{I}_2 + \bar{I}_2' &= (0.31)(1312.16) + (1312.16) + (1)(1312.16) - (1)(262.43) \\ &= 2768 \\ &\leq 3000.\end{aligned}$$

It is easy to see that the adjusted security constraint (14) will give the required level of security for any $c \geq 2768$.

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